MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

TMA 1301 – COMPUTATIONAL METHODS

(All sections / Groups)

6 MARCH 2017
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **8 pages** only with **4 Questions** and an **Appendix.**
2. Attempt ALL **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided, and **start each solution of a question on a new page.**
4. Show all steps.

Question 1

a) Given the polynomial function,

$$f(x) = 6x^2 - 7x + 3x^4 + 11 - 2x^3$$

(i) Turn the given function into ***nested form***.

[2 marks]

(ii) Calculate the value of f (0.7890) using (i) and **five-digit arithmetic** with rounding.

[2 marks]

(iii) Taking 9.392385 as an actual value of f (0.7890), find the **relative error** for the computed value from (ii). Write your answer correct to **seven** decimal places.

[1 mark]

b) Consider the following function,

$$g(x) = \cos x - \tan x$$

[Note: $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots$ and $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\ldots$]

(i) Rewrite the given function to avoid loss of significance by using **first three nonzero terms** in the Taylor series expansion.

[2 marks]

(ii) Calculate the value of $g(0.123)$ using (i) and **five-digit arithmetic** with rounding.

[2 marks]

(iii) Taking 0.8688 as an actual value of $g(0.123)$, find the **absolute error**.

[1 mark]

Continued.......

Question 2

[**Note:** For this question, use **four decimal places** for all the workings.]

a) Suppose $f(x)$ is a function on interval [0,1] with $f(x) = \sin x - 0.75$

(i) Perform **three** iterations using **Newton's method** to reach an approximated root P_3 of the equation $f(x) = 0$, starting with initial value $P_0 = 0$. **Show** the working steps.

[4 marks]

(ii) If the actual root of the equation $f(x) = 0$ is 0.8481, compute the **absolute error** of the approximated root.

[1 mark]

b) (i) Find the definite integral $\int_0^1 e^x \, dx$.

[1 mark]

(ii) Approximate $\int_0^1 e^x \, dx$ by using the **Trapezoidal Rule** with 5 points.

[3 marks]

(iii) Hence, compute the **absolute error** for the approximation from (ii) (Take the approximation of the answer (i) correct to four decimal places as the actual value).

[1 mark]

Continued.......

Question 3

a) Use row reduction technique to find an upper triangular **U** and lower triangular **L** in the **LU factorization** of the following matrix:

$$\begin{bmatrix} 2 & 4 & -4 \\ 1 & -4 & 3 \\ -6 & -9 & 5 \end{bmatrix}$$

[4 marks]

b) Construct the equations for w, x, y and z of the following linear system. Then compute the first iteration of the **Gauss-Seidel Method**

$$\begin{array}{rcrcrcrcr} 10w & - & x & + & 2y & & & = & 6 \\ -w & + & 11x & - & y & + & 3z & = & 25 \\ 2w & - & x & + & 10y & - & z & = & -11 \\ & & 3x & - & y & + & 8z & = & 15 \end{array}$$

Copy the following table into your **Answer Booklet** and **complete** it. Write your answers correct to **six** decimal places.

n	w	x	y	z
0	0	0	0	0
1				

[4 marks]

c) Find the **eigenvalues** for the following **matrix A**:

$$A = \begin{bmatrix} -4 & -17 \\ 2 & 2 \end{bmatrix}$$

[2 marks]

Continued.......

Question 4

[**Note:** For this question, use **four decimal places** for all the workings.]

a) The following table shows the population of Malaysia from year 2008 to 2015.

Year	2008	2009	2010	2011	2012	2013	2014	2015
Population (in million)	27.1	27.6	28.1	28.5	29.0	29.4	29.9	30.3

The population of Malaysia is assumed to have an exponential growth. The equation of an exponential function is given as $y = ae^{bx}$.

(i) The exponential function can be linearized in the form of $Y = A + bx$ by taking logarithm. Find the expressions for Y and A.

[1 mark]

(ii) Copy the following table into your **Answer Booklet** and **complete it**.

Year	x	y	x^2	Y	xY
2008	1	27.1			
2009	2	27.6			
2010	3	28.1			
2011	4	28.5			
2012	5	29.0			
2013	6	29.4			
2014	7	29.9			
2015	8	30.3			
	$\sum x =$		$\sum x^2 =$	$\sum Y =$	$\sum xY =$

[2 marks]

(iii) From (i) and (ii), find the exponential function of the form $y = ae^{bx}$ using the **method of least squares.**

[2 marks]

(iv) From (iii), estimate the population (in million) of Malaysia in **year 2017**.

[1 mark]

Continued.......

[**Note:** For this question, use **four decimal places** for all the workings.]

b) Given a function $f(x)=\dfrac{1}{1-x}$.

(i) Find the **second degree Mclaurin Polynomial** from the given $f(x)$.

[3 marks]

(ii) From (i), approximate $f(0.25)$.

[0.5 mark]

(iii) Find the **absolute error** for the approximation obtained in (ii).

[0.5 mark]

End of Exam Questions.

APPENDIX - FORMULAS

1. LOCATING ROOTS OF EQUATIONS

(A) Bisection Method

$$p_n = \frac{b_n + a_n}{2}$$

(B) Secant Method

$$p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$$

(C) Newton's Method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

2. NUMERICAL INTEGRATION

(A) Trapezoidal Rule

$$A_T = \int_a^b f(x)dx \approx \frac{h}{2}\{f(x_1) + f(x_{n+1}) + 2[f(x_2) + f(x_3) + \ldots + f(x_n)]\}.$$

(B) Simpson's Rule

$$A_S = \int_a^b f(x)dx \approx \frac{h}{3}\begin{Bmatrix} f(x_1) + f(x_{2n+1}) + 4[f(x_2) + f(x_4) + \ldots + f(x_{2n})] \\ + 2[f(x_3) + f(x_5) + \ldots + f(x_{2n-1})] \end{Bmatrix}$$

Continued...

2. NUMERICAL INTEGRATION (CONTINUED)

(C) Romberg Algorithm

$$R(0,0)=\frac{1}{2}(b-a)[f(a)+f(b)]$$

$$R(n,0)=\frac{1}{2}R(n-1,0)+h\sum_{k=1}^{2^{n-1}}f[a+(2k-1)h];\quad h=\frac{(b-a)}{2^n},\ n\ge 1$$

$$R(n,m)=R(n,m-1)+\frac{1}{4^m-1}[R(n,m-1)-R(n-1,m-1)],$$

where $n\ge 1,\ m\ge 1$

3. LEAST SQUARES PROBLEMS, INTERPOLATION AND POLYNOMIAL APPROXIMATION

(A) Linear Least Squares $y=a+bx$

$$a=\frac{\sum_{i=1}^{n}x_i^2\sum_{i=1}^{n}y_i-\sum_{i=1}^{n}x_iy_i\sum_{i=1}^{n}x_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2},\quad b=\frac{n\sum_{i=1}^{n}x_iy_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}$$

(B) Taylor Polynomial

$$P_n(x)=\sum_{k=0}^{n}\frac{1}{k!}f^{(k)}(a)(x-a)^k$$

(C) Lagrange Polynomial

$$P_n(x)=\sum_{i=0}^{n}f(x_i)L_i(x)$$

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